Computing Probabilities of Different Numbers of Casualties for Reentering Satellites with Large Casualty Areas

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The question at hand is how to compute the probability that a given number of people would become casualties from the reentry of a satellite with a single large casualty area. In order to make this computation, a number of simplifying assumptions must be clarified. These assumptions deal with how the population of the Earth is distributed, how casualty is determined, and how the reentry footprint will be calculated.

Population

The population files used for these calculations are from the Socioeconomic Data and Applications Center (SEDAC) at Columbia University. The data set used is the Gridded Population of the World, version 3 (GPWv3). This data set estimates the population in latitude/longitude grid positions on the Earth divided into 2.5×2.5 arc minute cells for reference years 1990-2015 in 5-year intervals. These cells are approximately 4.6 km long in the north-south direction. At the equator, the cells are also approximately 4.6 km wide in the east-west direction, but are narrower at more northerly and southerly latitudes.

The population estimated for each cell is often not a whole number, so for the purposes of the calculations outlined below, the population in each cell was rounded to the nearest integer. This alters the total global population count by a factor less than 0.1%, so it is not expected that this approximation will substantially change the conclusions.

Because there is no information (in this data set) on how the population is distributed within a cell, it is assumed that people are randomly distributed within each latitude/longitude cell.

Note that this model assumes nothing about sheltering – all humans are assumed to be exposed. Note also that there is no need to determine which cells are land cells and which cells are water, because the GPWv3 model assigns zero population to water cells. This does not take into account the number of people in boats or aircraft, but this should not substantially change the results.

We also assume that a cell is all land if it has any population at all. This may introduce some error for densely populated areas that are located next to large bodies of water. Some such cells could contain regions of both land and water. The population in such cells would not be distributed evenly throughout the cell, but only over the (unknown) fraction of the cell that is land area. The GPWv3 data set actually includes information on the fraction of each cell that contains land, but SEDAC has had some minor technical issues with that data set (that should be corrected in the near future), so we chose not to use it at this time. It is not expected that inclusion of this data will substantially change the results.

The Earth

The area of each latitude/longitude cell was computed by assuming a spherical Earth with radius 6378.135 km. In reality, there are slight differences between an ideal spherical Earth and the slightly oblate Earth on which the latitude/longitude grid is placed. However, the differences are small, and combined with the simplifying orbit assumptions (below), the spherical assumption should be adequate.

The area *A* (in square kilometers) for each latitude/longitude cell is computed as

$$A = (\alpha_2 - \alpha_1)\left(Sin(\delta_2) - Sin(\delta_1)\right)(6378.135\ km)^2$$

where α_1 is the longitude of the western edge of the cell, α_2 the longitude of the eastern edge, δ_1 is the latitude of the southern edge, and δ_2 is the latitude of the northern edge.

Note that the longitude angles must be converted to radians for this equation. Because the angular width in longitude of all cells is the same (2.5'), the $(\alpha_2 - \alpha_1)$ term will always be the same, or about 0.00072722 radians.

Casualty Area

For this study, the casualty area is assumed to be from a single large “object” - much larger than the size of a person, but much smaller than the area of a latitude/longitude cell. Thus, humans can be assumed to be points randomly distributed within each latitude/longitude cell, and they may or may not be within the casualty area. Likewise, the casualty area can be assumed to fall completely within a single cell (i.e., not straddling cells). It is also assumed to fall at a random position within the cell.

If a person is within the casualty area at the instant of the fall, that person is assumed to be a casualty. If the person is not within the casualty area, that person is not considered a casualty.

If an “object” with a casualty area *a* falls at a random location within a particular latitude/longitude cell, and we know the area of that cell *A* and the integer number of people *N* in that cell (assumed to be points randomly distributed throughout the cell), we are in a position to assess the probability that a particular number of people in that cell are casualties. If we envision the casualty area *a* as a sub-area within the much larger latitude/longitude cell, then the probability that out of *N* total people in the cell, *j* are within the casualty area is a simple binomial relationship :

$$b(j,N,a,A)=\binom{N}{j}\left(\frac{a}{A}\right)^{j}\left(1-\frac{a}{A}\right)^{N-j}=\frac{N!}{j!\,(N-j)!}\left(\frac{a}{A}\right)^{j}\left(1-\frac{a}{A}\right)^{N-j} \quad .$$

The probability that a given number of people *k* or more (where $k \leq N$) are within the casualty area is given by summing

$$B(k,N,a,A)=\sum_{j=k}^{N} b(j,N,a,A) \quad .$$

Orbit

The exact position where an object will reenter is not known with any precision weeks before the event. Because orbits generally precess (their ascending nodes change with time) and because the Earth rotates beneath the orbit, the reentry longitude can be assumed to be random. For circular (or nearly-circular) orbits the reentry can occur at any mean longitude position (any position along the orbit), so this can be assumed to be random as well.

The only orbital parameter of relevance is the inclination – the tilt of the orbit plane with respect to the Earth's equator. Depending on the inclination, a satellite spends different amounts of time over different latitude bands. In addition, a satellite orbits within a range of latitudes between a maximum (northernmost) and minimum (southernmost) latitude (note that southern latitudes are given negative values). The latitude range of a satellite is $-\mathrm{Sin}(i) \leq \mathrm{Sin}(\delta) \leq \mathrm{Sin}(i)$, where δ is the latitude and i is the inclination of the orbit. An extreme example would be an equatorial satellite with inclination 0º. Such a satellite would never pass overhead at any latitude other than the equator (δ = 0º).

Note that these conditions assume a perfect Kepler orbit, when in fact orbits experience a number of perturbations due to slight variations in the Earth's gravitational field. These variations are due to the fact that the Earth's globe is not a perfect sphere nor is the mass even distributed around it. However, these effects are minor and should not greatly affect the results.

The fraction of time an ideal Kepler orbit spends over a latitude band is given by the equation

$$f(i,\delta_1,\delta_2)=g(i,\delta_2)-g(i,\delta_1)$$

where

$$g(i,\delta) = +\frac{1}{2} \qquad if\ \ Sin(i) < Sin(\delta)$$

$$g(i,\delta) = \frac{1}{\pi} ArcSin\left(\frac{Sin(\delta)}{Sin(i)}\right) \quad if\ -Sin(i) \le Sin(\delta) \le Sin(i)$$

$$g(i,\delta) = -\frac{1}{2} \qquad if\ \ Sin(\delta) < -Sin(i)$$

and i is the inclination and δ is the latitude.

The probability that a satellite will fall in a particular latitude/longitude cell is proportional to the amount of time such an object spends over that cell during its orbit. Assuming that the object has no preferred longitude, the probability p of an orbiting object with a given inclination i falling within a particular latitude/longitude cell would be

$$p(i,\delta_1,\delta_2) = \left(g(i,\delta_2) - g(i,\delta_1)\right)\frac{(\alpha_2-\alpha_1)}{2\pi} = \left(g(i,\delta_2) - g(i,\delta_1)\right) \times 0.00011574 \quad .$$

The 0.00011574 term corresponds to the inverse of 8640 – the total number of cell divisions in the longitude direction.

Probability

We are now in a position to compute the total probability C that a satellite with a given inclination i and a single large casualty area a will have k or more casualties. This is done by summing over all M population cells. For each cell, we take the probability p that the satellite will fall in that cell multiplied by the probability B that a fall in that cell will produce k or more casualties.

$$C(i,a,k) = \sum_{m=1}^{M} p(i,\delta_{1m},\delta_{2m})\, B(k,N_m,a,A_m)$$

i – satellite inclination
a – satellite casualty area
k – lower limit of number of casualties
δ_{1m} – southern latitude of latitude/longitude cell m boundary
δ_{2m} – northern latitude of latitude/longitude cell m boundary
N_m – number of people in latitude/longitude cell m
A_m – area of latitude/longitude cell m

Note, A_m and a need to be in the same units (e.g., square kilometers).